Aufgabe 1

a) $\underline{Z}_{11} = \frac{\underline{U}_1}{\underline{I}_1} = j\omega L_1 + j\omega L_3$

$\underline{Z}_{21} = \frac{\underline{U}_2}{\underline{I}_1} = j\omega L_3 + j\omega L_2$

$\underline{Z}_{12} = \frac{\underline{U}_1}{\underline{I}_2} = j\omega L_3$

$\underline{Z}_{22} = \frac{\underline{U}_2}{\underline{I}_2} = j\omega L_2 + j\omega L_3$

$$[\underline{Z}] = \begin{pmatrix} j(\omega L_1 + \omega L_3) & j\omega L_3 \\ j\omega L_3 & j(\omega L_2 + \omega L_3) \end{pmatrix}$$

b) $\det \underline{Z} = j^2(\omega^2 L_1 L_2 + \omega^2 L_1 L_3 + \omega^2 L_2 L_3 + \omega^2 L_3^2) - j^2\omega^2 L_3^2$

$= -1(\ldots)$

c) $$\underline{Y}_1 = \begin{pmatrix} \frac{\underline{Z}_{22}}{\det \underline{Z}} & \frac{-\underline{Z}_{12}}{\det \underline{Z}} \\ \frac{-\underline{Z}_{21}}{\det \underline{Z}} & \frac{\underline{Z}_{11}}{\det \underline{Z}} \end{pmatrix} = \begin{pmatrix} \frac{j\omega(L_1+L_2)}{j^2\omega^2(L_1L_2+L_1L_3+L_2L_3+L_3^2)} & \frac{-j\omega L_3}{j^2\omega^2(\ldots)} \\ \frac{-L_3}{j\omega(L_1L_2+L_1L_3+L_2L_3+L_3^2)} & \frac{L_2+L_3}{j\omega(L_1L_2+L_1L_3+L_2L_3+L_3^2)} \end{pmatrix}$$

c) $$\underline{Y}_2 = \begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{pmatrix}$$

$\underline{Y}_{11} = \frac{\underline{I}_1}{\underline{U}_1} = \frac{1}{R_1} + \frac{1}{R_2}$ $\qquad$ $\underline{Y}_{12} = \frac{\underline{I}_1}{\underline{U}_2} = -\frac{1}{R_2}$

$\underline{Y}_{21} = \frac{\underline{I}_2}{\underline{U}_1} = -\frac{1}{R_2}$ $\qquad$ $\underline{Y}_{22} = \frac{\underline{I}_2}{\underline{U}_2} = \frac{1}{R_2} + \frac{1}{R_3}$

$$\det \underline{Y} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{1}{R_2^2}$$

$$= \frac{1}{R_1R_2} + \frac{1}{R_1R_3} + \frac{1}{R_2^2} + \frac{1}{R_2R_3} - \frac{1}{R_2^2}$$

$$= \frac{1}{R_1R_2} + \frac{1}{R_1R_3} + \frac{1}{R_2R_3} = \frac{R_1+R_2+R_3}{R_1R_2R_3}$$

$$\underline{Z} = \frac{\left(\frac{1}{R_3} + \frac{1}{R_2}\right)\left(R_1R_2R_3\right)\frac{1}{}}{R_1+R_2+R_3}$$

$$\underline{\underline{Z}} = \begin{pmatrix} \frac{R_1R_2 + R_1R_3}{R_1+R_2+R_3} & \frac{R_1R_3}{R_1+R_2+R_3} \\ \frac{R_1R_3}{R_1+R_2+R_3} & \frac{R_2R_3+R_1R_3}{R_1+R_2+R_3} \end{pmatrix}$$

f) $\underline{\underline{Z}} = \underline{\underline{Z}}_1 + \underline{\underline{Z}}_2 =$

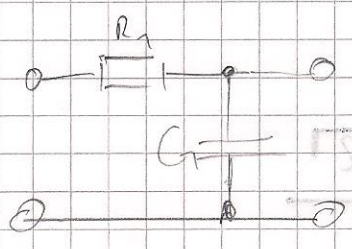
$$\begin{pmatrix} \frac{R_1R_2+R_1R_3}{R_1+R_2+R_3} + j\omega(L_1+L_2) & \frac{R_1R_3}{R_1+R_2+R_3} + j\omega L_3 \\ \frac{R_1R_3}{R_1+R_2+R_3} + j\omega L_3 & \frac{R_2R_3+R_1R_3}{R_1+R_2+R_3} + j\omega(L_2+L_3) \end{pmatrix}$$

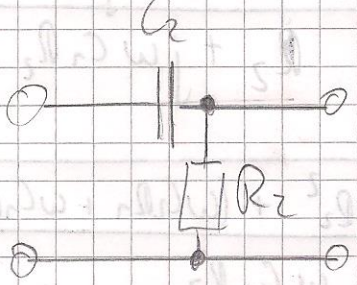
e) $\underline{\underline{Y}} = \underline{\underline{Y}}_1 + \underline{\underline{Y}}_2 =$

$$\begin{pmatrix} \frac{L_1+L_2}{j\omega(L_1L_2+L_1L_3+L_2L_3)} + \frac{1}{R_1} + \frac{1}{R_2} & -\frac{L_3}{j\omega(L_1L_2+L_1L_3+L_2L_3)} - \frac{1}{R_2} \\ -\frac{L_3}{j\omega(L_1L_2+L_1L_3+L_2L_3)} - \frac{1}{R_2} & \frac{L_2+L_3}{j\omega(L_1L_2+L_1L_3+L_2L_3)} + \frac{1}{R_2} + \frac{1}{R_3} \end{pmatrix}$$

Aufgabe 2

a)





b)

$\underline{z}_{11} = \frac{\underline{U}_1}{\underline{I}_2} = R_1 + j\omega C_1$

$\underline{z}_{11} = R_2 + j\omega C_2$

$\underline{z}_{21} = \frac{\underline{U}_2}{\underline{I}_1} = j\omega C_1$

$\underline{z}_{21} = \frac{\underline{U}_2}{\underline{I}_1} = R_2$

$\underline{z}_{12} = j\omega C_1$

$\underline{z}_{12} = R_2$

$\underline{z}_{22} = j\omega C_1$

$\underline{z}_{22} = R_{22}$

$[\underline{z}_1] = \begin{pmatrix} R_1 + j\omega C_1 & j\omega C_1 \\ j\omega C_1 & j\omega C_1 \end{pmatrix}$

$[\underline{z}_1] = \begin{pmatrix} R_2 + j\omega C_2 & R_2 \\ R_2 & R_2 \end{pmatrix}$

$\det \underline{z} = j\omega C_1 R_1 - \omega^2 C_1^2 + \omega^2 C_1^2$
$= j\omega C_1 R_1$

$\det \underline{z} = R_2^2 + j\omega C_2 R_2 - R_2^2$
$= j\omega C_2$

$[\underline{A}_1] = \begin{pmatrix} \frac{R_1 + j\omega C_1}{j\omega C_1} & R_1 \\ \frac{1}{j\omega C_1} & 1 \end{pmatrix} \begin{pmatrix} \frac{R_2 + j\omega C_2}{R_2} & j\omega C_2 R_2 \\ \frac{1}{R_2} & 1 \end{pmatrix} = [A_c]$

$A_{11,\text{kompl. Sch.}} = \frac{\underline{U}_2}{\underline{U}_1} = \frac{R_1 + j\omega C_1}{j\omega C_1} \cdot \frac{R_2 + j\omega C_2}{R_2} + \frac{R_1}{j\omega C_1 \cdot R_2} = H(j)$

$\underline{H}(j) = \frac{R_1 R_2 + R_1 j\omega C_2 + R_2 j\omega C_1 + \omega^2 C_1 C_2}{R_2 j\omega C_1} + \frac{R_1}{R_2}$

$$H(f) = \frac{R_1 R_2 + j\omega C_2 R_1 R_2 + j\omega C_1 R_2 + R_1 j\omega C_1}{j\omega C_1 R_2}$$

$$|H(f)| = \frac{\sqrt{R_1 R_2^2 + (\omega C_2 R_1 + \omega C_1 R_2 + \omega R_1 C_1)^2}}{\omega C_1 R_2}$$