**9. Schwerpunkt:** Differenzieren

0.9.1.T Bilden Sie die erste Ableitung der Funktionen nach x.

a)

$$y = f(x) = 3x^4 + 4x^2 - \frac{1}{5x}$$
$$f'(x) = 12x^3 + 8x + \frac{1}{5x^2}$$

b)

$$y = f(x) = \frac{1}{2x-1} + \frac{5}{(x-3)^2}$$
$$f'(x) = -\frac{2}{(2x-1)^2} - \frac{10}{(x-3)^3}$$

c)

$$y = f(x) = \sqrt{x} + (3x)^{5/2}$$
$$f'(x) = \frac{1}{2\sqrt{x}} + \frac{15}{2}(3x)^{3/2}$$

d)

$$y = f(x) = x\sqrt{2x+5}$$
$$f'(x) = \sqrt{2x+5} + \frac{x}{\sqrt{2x+5}}$$

e)

$$y = f(x) = \sin(2x)\cos(3x)$$
$$f'(x) = 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x)$$

f)

$$y = f(x) = \ln(3x) - \ln(2x)$$
$$f'(x) = \frac{3}{3x} - \frac{2}{2x} = 0$$

g)

$$y = f(x) = \frac{\sin(2x)}{\cos(3x)}$$
$$f'(x) = \frac{2\cos(2x)\cos(3x) + 3\sin(2x)\sin(3x)}{\cos^2(3x)}$$

h)

$$y = f(x) = \ln(3x)\cdot\ln(2x)$$
$$f'(x) = \frac{1}{x}\ln(2x) + \frac{1}{x}\ln(3x) = \frac{1}{x}\ln\left(6x^2\right)$$

0.9.2.T Bilden Sie die erste Ableitung der Funktion nach der Unabhängigen

a)

$$u = f(t) = \hat{U}e^{-2t}\sin(5t)$$
$$\dot{u} = f'(t) = \hat{U}\left(-2e^{-2t}\sin(5t) + 5e^{-2t}\cos(5t)\right)$$
$$= \hat{U}e^{-2t}\left(5\cos(5t) - 2\sin(5t)\right)$$

b)

$$u = f(t) = \hat{U}e^{-2t+1}\sin(3t+30^\circ)$$
$$\dot{u} = f'(t) = \hat{U}\left(-2e^{-2t+1}\sin(3t+30^\circ) + 3e^{-2t+1}\cos(3t+30^\circ)\right)$$
$$= \hat{U}e^{-2t+1}\left(3\cos(3t+30^\circ) - 2\sin(3t+30^\circ)\right)$$

c)

$$i = f(t) = A\cdot t\cdot\cos(\omega t+\varphi)$$
$$\frac{di}{dt} = f'(t) = A\left(\cos(\omega t+\varphi) - \omega t\cdot\sin(\omega t+\varphi)\right)$$

d)

$$x = f(y) = \sqrt{y^2-3y+8}$$
$$\frac{dx}{dy} = f'(y) = \frac{2y-3}{2\sqrt{y^2-3y+8}}$$

e)

$$v = f(w) = \frac{\ln(w+1)}{\ln(w+2)}$$
$$\frac{dv}{dw} = f'(w) = \frac{\frac{1}{w+1}\ln(w+2) - \frac{1}{w+2}\ln(w+1)}{\left(\ln(w+2)\right)^2}$$

f)

$$a = f(b) = \frac{b^2-4}{b+1}$$
$$\frac{da}{db} = f'(b) = \frac{2b(b+1) - \left(b^2-4\right)}{(b+1)^2} = \frac{2b^2+2b-b^2+4}{b^2+2b+1} = \frac{b^2+2b+4}{b^2+2b+1}$$

g)

$$y = f(x) = x\cdot e^{2x}\cdot\sin(3x)$$
$$\frac{dy}{dx} = f'(x) = \left(e^{2x}+2xe^{2x}\right)\sin(3x) + 3xe^{2x}\cos(3x)$$
$$= e^{2x}\left(\sin(3x) + 2x\sin(3x) + 3x\cos(3x)\right)$$

NR :

$$u = x\cdot e^{2x} \qquad u' = e^{2x} + 2xe^{2x}$$

h)

$$y = f(t) = \hat{Y}\cdot\sin^2(2t-\pi)$$
$$\dot{y} = f'(t) = 4\hat{Y}\sin(2t-\pi)\cos(2t-\pi)$$

0.9.3.T An welchen Stellen hat die Funktion eine waagerechte Tangente ?

a)

$$y = f(x) = x \cdot e^{-3x}$$

$$f'(x) = e^{-3x}(1 - 3x) = 0$$

$e^{-3x}$ wird nicht 0

$$1 - 3x = 0$$

$$x = \frac{1}{3}$$

b)

$$y = f(x) = e^{-(x-3)^2}$$

$$f'(x) = -2(x-3)e^{-(x-3)^2} = 0$$

$$x - 3 = 0$$

$$x = 3$$

c)

$$y = f(x) = x^2 \cdot e^{-3x}$$

$$f'(x) = e^{-3x}(2x - 3x^2) = 0$$

$$x(-3x + 2) = 0$$

$$x_1 = 0 \qquad -3x_2 + 2 = 0$$

$$x_2 = \frac{2}{3}$$

d)

$$y = f(x) = \frac{1}{2}(e^{3x} + e^{-3x})$$

$$f'(x) = \frac{3}{2}(e^{3x} - e^{-3x}) = 0$$

$$e^{3x} - e^{-3x} = 0$$

$$e^{3x} = e^{-3x}$$

$$3x = -3x$$

$$x = 0$$

e)

$$y = f(x) = \frac{1}{(x+4)^2}$$

$$f'(x) = \frac{-2}{(x+4)^3} = 0$$

wird im unendlichen 0

keine waagerechten Tangenten

f)

$$y = f(x) = \frac{1}{3}x^3 + x^2 + x + 1$$

$$f'(x) = x^2 + 2x + 1 = 0$$

$$x_{1/2} = -1$$